QAA MEMBERSHIP

Subject Benchmark Statement

Mathematics, Statistics and Operational Research

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About this Statement

This document is a QAA Subject Benchmark Statement for Mathematics, Statistics and Operational Research that defines what can be expected of a graduate in the subject, in terms of what they might know, do and understand at the end of their studies. Subject Benchmark Statements are an established part of the quality assurance arrangements in UK higher education, but not a regulatory requirement. They are sector-owned reference points, developed and written by academics on behalf of their subject. Subject Benchmark Statements also describe the nature and characteristics of awards in a particular subject or area. Subject Benchmark Statements are published in QAA's capacity as an expert quality body on behalf of the higher education sector. A summary of the Statement is also available on the QAA website.

Key changes from the previous Subject Benchmark Statement include:

- a revised structure for the Statement, which includes the introduction of cross-cutting themes of:
  - equality, diversity, and inclusion
  - accessibility and the needs of disabled students
  - education for sustainable development
  - employability, entrepreneurship and enterprise education
- a comprehensive review updating the context and purposes of Mathematics, Statistics and Operational Research (MSOR) including course design and content in order to inform and underpin the revised benchmark standards.

How can I use this document?

Subject Benchmark Statements are not intended to prescribe any particular approaches to teaching, learning or assessment. Rather, they provide a framework, agreed by the subject community, that forms the basis on which those responsible for curriculum design, approval and update can reflect upon a course, and its component modules. This allows for flexibility and innovation in course design while providing a broadly accepted external reference point for that discipline.

They may also be used as a reference point by external examiners in considering whether the design of a course and the threshold standards of achievement are comparable with those of other higher education providers. They also support professional, statutory and regulatory bodies (PSRBs) with the academic standards expected of students.
You may want to read this document if you are:

- involved in the design, delivery and review of courses in Mathematics, Statistics and Operational Research
- a prospective student thinking about undertaking a course in Mathematics, Statistics and Operational Research
- an employer, to find out about the knowledge and skills generally expected of Mathematics, Statistics and Operational Research graduates.

Relationship to legislation

The responsibility for academic standards lies with the higher education provider which awards the degree. Higher education providers are responsible for meeting the requirements of legislation and any other regulatory requirements placed upon them by their relevant funding and regulatory bodies. This Statement does not interpret legislation, nor does it incorporate statutory or regulatory requirements.

The regulatory status of the Statement will differ depending on the educational jurisdictions of the UK. In England, Subject Benchmark Statements are not sector-recognised standards as set out under the Office for Students' regulatory framework. However, they are specified as a key reference point, as appropriate, for academic standards in Wales under the Quality Assessment Framework for Wales and in Scotland as part of the Quality Enhancement Framework. Subject Benchmark Statements are part of the current quality arrangements in Northern Ireland. Because the Statement describes outcomes and attributes expected at the threshold standard of achievement in a UK-wide context, many higher education providers will use them as an enhancement tool for course design and approval, and for subsequent monitoring and review, in addition to helping demonstrate the security of academic standards.

Additional sector reference points

Higher education providers are likely to consider other reference points in addition to this Statement in designing, delivering and reviewing courses. These may include requirements set out by PSRBs and industry or employer expectations. QAA has also published Advice and Guidance to support the Quality Code, which will be helpful when using this Statement - for example, in course design, learning and teaching, external expertise and monitoring and evaluation.

Explanations of unfamiliar terms used in this Subject Benchmark Statement can be found in QAA's Glossary. Sources of information about other requirements and examples of guidance and good practice are signposted within the Statement where appropriate.

1 Context and purposes of a Mathematics, Statistics and Operational Research degree

1.1 For the purposes of this document, the term 'MSOR' (Mathematics, Statistics and Operational Research) should be understood to be more than the disjoint union of its three named elements and instead be understood to encompass a wide variety of combinations of the various fields. There is considerable overlap between mathematics, statistics and operational research, and the modern practitioner of any of the individually named fields is likely to be familiar with elements of the others. Indeed, many MSOR courses and modules, such as those in interdisciplinary programmes in data science, exploit this commonality to assemble a coherent educational provision that draws together elements of mathematics, statistics, operational research and other disciplines in order to address particular problems.

1.2 Although, for the purposes of this Subject Benchmark Statement, we refer to MSOR as a coherent corpus of knowledge and skills in its own right, it is perhaps useful to briefly describe each of the named subfields.

1.3 Mathematics is a major intellectual subject in its own right, with a history that extends back through various cultures, both ancient and recent. It has its roots in the systematic development of methods to solve practical problems in areas such as surveying, mechanical construction and commerce. The subject evolved with the realisation that such methods, when stripped of the details of the particular situation, had a wide range of applications and highlighted the essential common characteristics of many different problems. Therefore, generalisation and abstraction became important features of the subject. This abstraction allows mathematicians to find deeper relationships within the patterns than could otherwise have been found from observation or unaided reasoning. This then enables common solutions to be found to problems that would otherwise have seemed unrelated.

1.4 Today, mathematics is a subject in which strict logical deductions are used to draw conclusions that follow with certainty from a given set of assumptions. These assumptions may be abstractions of fundamental concepts such as number, shape or symmetry, or they may be simplified models of real-world systems. While the mathematics of earlier times still remains relevant, it is now only a small part of an ever-expanding and dynamic subject.

1.5 Statistics is the scientific discipline of collecting, analysing, interpreting and presenting data, particularly in situations where there is random variation in the data caused by, for example, sampling variation or observational errors. At its heart is probability theory, a branch of mathematics which formalises concepts such as probability distributions and stochastic processes based on an axiomatic system. The modern field of probability and statistics primarily began in seventeenth century Europe through the study of games of chance but some concepts were explored much earlier; for example, Arab mathematicians wrote about combinatorics in the eighth century and the Roman Empire routinely compiled official statistics from census data.

1.6 Today, statistical models are built to represent relationships within data in order to test hypotheses, describe associations or forecast/predict unobserved values, while probability modelling is at the heart of areas such as epidemiological modelling and machine learning. Statistical techniques are also a key part of the rapidly growing interdisciplinary field of data science where they are used to gain insights from the large amounts of data generated in this digital age, including non-numerical forms of data such as images and text. While statistics and probability are mathematical sciences because of the rigorous theory that underpins them, the subjects are routinely applied in a wide range of contexts, such as analysing data from scientific experiments or supporting decision-making in business management and government. Consequently, a key aspect of applied statistics is communicating conclusions to non-experts.

1.7 Operational research is a more recent subject, beginning during the twentieth century, and many of its origins are to be found in the organisation of activities during the Second World War. The subject ranges from complex optimisation procedures with significant mathematical underpinning to non-mathematical but academically rigorous problem-structuring methods and techniques for informing decision-making and strategy development. It finds important applications in many sectors of society, in particular, business, health and social services, and the armed forces.

1.8 The subject area of analytics has become increasingly associated with operational research in recent years and operational research has become one of the key quantitative management approaches of modern times. Although the name 'operational research' is generally well understood, a number of providers use other titles for courses in this area, such as 'management science'. Titles of this sort often indicate very application-focused courses, perhaps with relatively little mathematical content. Such courses, by virtue of their design, might not fall entirely within this Subject Benchmark Statement.

1.9 The influence of MSOR continues to grow significantly, within and outside traditional science, technology, engineering and mathematics (STEM) disciplines, and both in research and in taught courses. MSOR is also distinctive in the extent to which the subject is taught by subject experts to non-specialists as service courses. Some programmes of study in other disciplines, or individual courses within those programmes, are sufficiently dependent on MSOR that this Subject Benchmark Statement is directly relevant, such as mathematics for engineering.

1.10 MSOR course curricula form a broad spectrum of styles. To meet students' needs, courses may combine or focus more on one of these styles. At one end of the spectrum are 'theory-based courses' that are concerned with the way in which theory establishes general propositions leading to methods and techniques which can then be applied to a range of problems. At the other end are 'practice-based courses' that cover the understanding and application of results, methods and techniques to a variety of situations in different contexts. The subject's interdisciplinary nature means courses may apply theory and practice to areas both within and outside MSOR. Providers should take steps to ensure that all MSOR graduates, whatever the nature of the course they have studied, are able to fully identify as professional MSOR practitioners.

Purposes and characteristics of an MSOR degree

1.11 The study of MSOR develops analytical creativity and explores relationships among abstract concepts without necessarily considering potential real-world counterparts. Many years later even solely academic research can have ground-breaking impact in new and developing fields. Equally, MSOR can focus on solving problems with immediate practical applications. The distinction between theoretical and applied approaches can be blurred with shared techniques that examine patterns and relationships, with differences only emerging through purpose.

1.12 As a result of the breadth of MSOR as a discipline, each higher education provider awarding qualifications in MSOR defines the content, nature and organisation of its courses and modules. Consequently, MSOR courses offered by individual providers will have their own particular characteristics.

1.13 MSOR courses include a wide variety of configurations, including foundation years, apprenticeships, single honours, joint honours, integrated master's and postgraduate taught master's degrees. Many providers find each plays an important and distinctive role.

- Integrated master's courses generally include aspects of MSOR in greater depth and/or breadth than bachelor's courses and better prepare students for postgraduate research studies or employment. They typically include a substantial project. Some MSOR-led integrated master's courses have a focus in particular areas of application.
- Postgraduate taught master's courses have a range of distinct purposes. Some are conversion courses, allowing graduates from a broad range of other disciplines to retrain in MSOR. Others allow students to further extend the depth of specialised study, particularly as preparation for postgraduate research.
- Foundation years enable applicants to develop their foundational knowledge before progressing onto an honours degree course, either on standalone programmes or via direct progression.
- Some providers offer courses where learners spend a year, or shorter periods, in a supervised professional placement (see paragraph 2.28).

1.14 MSOR courses are an intellectual pursuit that develops wide-ranging academic and transferable skills, open up a range of further study opportunities, and provide an excellent route to employment. MSOR graduates have a wide choice of careers available to them.

1.15 MSOR has important general characteristics which pervade the culture of the discipline, including an underpinning in abstract and logical reasoning and a need for accuracy in numerical work and symbolic manipulation. As these occur throughout the discipline, time is needed to consolidate learning when developing and practising discipline skills.

1.16 It is an inherent characteristic of the subject that an individual student's performance may vary greatly over different modules. This non-uniform profile of attainment is a characteristic feature of MSOR. See also paragraph 3.52 on Assessment.

1.17 An important characteristic of the discipline is the cumulative nature of the subject; modules often require essential background knowledge and have strict formal prerequisites. It follows that it is quite normal, and often necessary, to teach very similar subject matter in different years. For example, identical material taught to single honours students in year one might be taught to joint-honours students in later years. Students on a conversion master's course might well need to learn material encountered much earlier by specialists. While greater general academic maturity might be expected from learners in subsequent years of a course there are no shortcuts to prerequisites. See also paragraph 2.12 on Progression.

1.18 MSOR graduates are well prepared to undertake further study in various subject areas, including, but not limited to, MSOR, and are highly employable. Employment opportunities may draw on explicit MSOR skills or utilise the skills developed in areas less directly related to their subject domain. An extensive source of information is available to students and graduates of MSOR courses from the Maths Careers website.

1.19 As described in both the 2012 report Measuring the Economic Benefits of Mathematical Science Research in the UK and the 2018 report The Era of Mathematics reviewing knowledge exchange in the mathematical sciences, MSOR and mathematical sciences more broadly make a vast, quantifiable contribution to the UK economy and society. For example, the gross value added by mathematical sciences research in 2010 was over 40% of the UK total. MSOR degrees are key to establishing a workforce to maintain and drive forward these societal and economic benefits.

1.20 As of 2023 the following societies offer professional recognition schemes based on the nature of the degree programme, an individual's education, an individual's work experience or a combination of these.

- The Institute of Mathematics and its Applications (IMA) accredits courses where they meet its requirements for graduates to attain its Chartered Mathematician status.
- The Royal Statistical Society (RSS) accredits courses where they meet its requirements for graduates to attain its Graduate Statistician status. The RSS also accredit individual modules that teach statistical literacy, awarding them the RSS Quality Mark.
- The IMA, RSS and Operational Research Society (ORS) all operate individual professional recognition schemes based on either the nature of the individual's education, engagement with continued professional development, their work experience or a combination of these.
- The Institute and Faculty of Actuaries (IFoA) accredits courses and modules, with accreditation providing students with exemption from certain IFoA examinations.

Equality, diversity and inclusion

1.21 Equality, diversity and inclusion (EDI) is essential for the health of MSOR, and it is important that the discipline encourages inclusivity and access to ensure learners are attracted from diverse backgrounds, that the curriculum and environment enable them to succeed in their studies, and that the subject is enriched by input from diverse practitioners.

1.22 Students come to MSOR courses with different backgrounds, aspirations, expectations and academic experiences. As such, all students benefit from developing their knowledge and understanding within an inclusive learning environment providing rich opportunities for academic, pastoral and well-being support which recognises this diversity. This would include the provision of suitable support for students with different needs and varying pre-university experiences, as outlined in more detail below.

1.23 Values of EDI should permeate the curriculum and every aspect of the learning experience to ensure the diverse nature of society in all its forms is evident. MSOR providers should reflect on their curricula and processes to ensure that no group is disadvantaged or othered; for example, by reflecting on how policies and practices around delivery, admission and assessment might adversely impact on certain subgroups within the student cohort. EDI aspects of student engagement and achievement should be monitored, and actions formed to ensure equity.

1.24 MSOR providers throughout the UK are committed to championing EDI, with several possessing Athena SWAN awards and/or providing publicly available information describing their commitment to EDI and initiatives in this domain. The London Mathematical Society (LMS) runs a Good Practice Scheme supporting women’s careers in MSOR in higher education and many providers work to these standards.

1.25 There is active discussion in higher education on the topic of decolonising the curriculum, and this topic is relevant to MSOR. Though there is no suggestion of omitting or censoring core discipline content, MSOR subjects are not neutral nor detached from society, and explicit reflection on the history and practice of MSOR knowledge generation can be useful in attempting to avoid unconscious biases in course design and delivery. While presenting the positive contributions that MSOR disciplines have made, provision might also reflect the fact that such contributions sometimes arose from cultural contexts with issues that would now be considered problematic. For example, some early ideas in statistics were motivated by their proposers’ support for eugenics, some astronomical data were collected on plantations by enslaved people, and, historically, some mathematicians have recorded racist or fascist views or connections to groups such as the Nazis. Further, there are historical and ongoing issues around power dynamics and gatekeeping in both access to and generation of MSOR knowledge. Providers should review their practices, in line with paragraph 1.23, with an explicit focus on equity.

1.26 It is highly desirable that students encounter a wide range of role models within higher education. This is particularly important given the well-known gender imbalance in the subject (the 2015 Council for Mathematical Sciences report on The Mathematical Sciences People Pipeline contains data on the demographic imbalances), retention and awarding gaps, and the strong focus of curricula on the historical work of white Western males. There is a need for inclusive language and scenarios in all publicity and teaching material, and for courses to be informed by the student voice and taught in a way which makes the resources meaningful to all students and with topics and examples which have relevance to a wide range of people.

1.27 EDI has implications for all aspects of provision. For example, providers might give consideration to good practice in areas such as the following.

Environment - good practice here could include:

- making efforts to present visible diverse role models, attracting diverse staff in all roles and ensuring a suitable working and study environment for all; for example, having an understanding of caring responsibilities for staff, which considers the timing of meetings and other practical arrangements
- provision of a suitable study environment for all with accessible facilities and equal access to specialist software and other technology
- being responsive to the student voice and actively soliciting suggestions to enhance provision.

Recruitment - good practice here could include:

- admissions decisions that acknowledge the variety of qualifications applicants might have, and consider the provision of opportunities for those without A Level Mathematics or equivalent (for example, via foundation years) or Further Mathematics
- providers recruiting from a variety of schools and colleges, promoting diverse role models and advertising the support they offer.

Delivery - good practice here could include:

- a timetable which considers the needs of students with commitments such as childcare and other caring responsibilities, religious beliefs and work, and ensures that suitable teaching spaces appropriate for all are allocated
- provision of materials that are accessible, written in inclusive language, with examples and scenarios which are appealing to all and relevant beyond the UK, recommending textbooks and other resources which conform to modern expectations regarding EDI
- the use of a variety of teaching methods to support learners with differing needs
- the use of tools like recording and electronic provision of resources to support those who may not be able to attend all lectures, or who may use these to learn more effectively (see paragraph 1.37)
- taking care over, for example, the assignment of students for groupwork.
- teaching staff being available to support students and possibly a university-wide mathematics and statistics support provision (see paragraph 3.33).

Curriculum - good practice here could include:

- a curriculum that presents a multicultural and contextualised view of MSOR, informed by the student voice
- where possible, a curriculum that presents the work of a diverse group of MSOR practitioners
- making students aware of problematic issues in the development of the MSOR content they are being taught
- embedding, within the MSOR curriculum, education in both general EDI matters and those that are specifically relevant to MSOR, such as the need to consider diversity in data collection and analysis.

Employment skills - good practice here could include:

- a curriculum that delivers career skills, balancing the needs of students who plan to go into graduate careers on completion of their degree and those aspiring to further study
- strategies designed to make teamwork inclusive, addressing issues such as norms regarding decision-making, attitudes to authority, positioning within a team, past experiences of task designation, reaching group decisions, and geopolitical issues.

Assessment - good practice here could include:

- the availability of a variety of assessment methods, and acknowledgement of the difficulties that students from different academic, social or cultural backgrounds may face with some forms of assessment
- monitoring data on attainment and taking actions to address the issues identified
- making reasonable adjustments where appropriate (see paragraph 1.31)
- design of assessments and assessment schedules giving consideration to students' mental well-being, taking care to minimise the stress these cause to students.

Support - good practice here could include:

- making support available for all students, including disabled students and those from diverse backgrounds or different cultures
- an inclusive culture across the institution, including internal workshops and training sessions to raise the awareness of MSOR teaching staff of issues relating to EDI.

Accessibility

1.28 In order to ensure an accessible course experience, providers should anticipate their learners' needs and proactively ensure that these are met. Providing institutions should support MSOR staff to overcome discipline-specific barriers to accessibility (see paragraphs 1.34 to 1.36) that can significantly increase staff time creating accessible resources and content. Provider good practice in this area may include support staff with disciplinary and technology knowledge, MSOR-accessibility-specific training, consideration of MSOR needs in policymaking, or ensuring learning tools adequately meet the needs of MSOR disciplines.

1.29 Providers should consider the challenges and barriers that need to be overcome to successfully empower their students to be full participants in their own education and academic communities. The diverse nature of MSOR courses and individual modules within courses means these challenges and barriers should be considered both at course level and within modules to create a coherent and consistent approach to accessibility.

1.30 In addition, it is imperative that providers anticipate the discipline-specific digital, physical and pedagogical accessibility needs of disabled students, embedding accessibility in a manner that minimises or eliminates the need for reasonable adjustments.

1.31 Reasonable adjustments respond to both the individual and the discipline, and provide authentic participation in all course activities. Where reasonable adjustments are necessary for individual students, providers may need to work with internal or external specialist disability advisory services to provide them.

1.32 Embedded accessibility has benefits for all students. In addition, it typically further benefits particular groups of students, such as those with caring responsibilities, those working part-time, those with a temporary impairment due to short-term illness or injury, and those experiencing acute mental distress.

1.33 Providers should seek to ensure that their courses are well curated. Well-curated courses benefit all students by promoting good mental health and well-being and reduce the administrative burden and cognitive load on disabled students. Well-curated courses include clearly communicated learning opportunities, assessments that are appropriately spaced and particular support when concepts and skills are initially introduced.

1.34 Distinctive discipline characteristics that may require specific attention include:

- the use of a wide range of special symbolism which is essential to understanding the subject matter, and special notation used by all MSOR specialists to communicate effectively and unambiguously
- traditional practices in teaching MSOR, for example, presenting a technical argument carefully at a board, which have evolved and are retained for very sound reasons: the highly compressed nature of the concepts and relationships encoded in mathematical notation needs to be discussed very carefully, sometimes symbol by symbol, with extended arguments presented
- discussion of mathematical arguments is often highly non-linear, for example regularly referring backwards to previous statements in precise ways
- the essential role played by diagrams and pictures in MSOR.

1.35 Specialist technology for MSOR is developing rapidly and continues to remove barriers to access. There have been significant positive advances in digital accessibility throughout this century. Further technological changes can be expected for the foreseeable future; for example, developing mathematical arguments by writing on a tablet can offer advantages over boards in terms of visibility in the lecture room, handwriting recognition, and clarity of captured content.

1.36 Mainstream adaptive technologies, for example learning management systems, screen readers and automatic captioning software, often do not effectively support MSOR content. To mitigate this deficiency:

- providers should acknowledge the special needs of MSOR and the deficiencies in many adaptive features of mainstream technology by providing additional resources which may not be needed by subjects where the medium is primarily text based
- academic staff will require specialist technical support to solve particular digital accessibility challenges
- staff and students are likely to need special hardware and software to better access MSOR content.

Consequently, creativity may be required to address challenges which, at the time of writing, remain.

1.37 Including elements of online, hybrid or blended learning across courses can significantly increase access for students, as can, where appropriate, the provision of captured content from in-person sessions. Associated enabling technology is required to capture the live display of extended written material and ensure accurate captions. Within a physical venue, consideration of the visibility of boards and other display equipment is essential.

1.38 Those in support roles, including scribes, notetakers, readers and communication support workers, need to have an understanding of MSOR terminology and symbolism since even minor symbolic differences can radically change the intended meaning.

1.39 Some teaching and learning activities are found relatively infrequently in MSOR courses. In such instances providers may take advice from other disciplines. This includes experimental mathematics and fieldwork which, where it exists, allows students to benefit from local opportunities.

1.40 A flexible course pedagogy provides students with a variety of learning opportunities, alternative ways to acquire knowledge and time to consolidate material. For instance, where material is delivered by a traditional lecture, the advanced provision of accessible lecture notes complemented by in-person delivery of the material which is recorded may provide such variety.

1.41 Implicit MSOR course requirements, such as social and cultural behaviours assumed without direct instruction, are context dependent, where a specific task or situation can change expectations. These can be additional barriers for disabled students. Examples may include appropriate computational accuracy, study expectations, organising independent study and teamworking skills. All students will benefit from, and disabled students may require, explicit instructions on these traditionally implied course norms.

1.42 Assessment design should anticipate and remove potentially irrelevant or context-dependent barriers that are irrelevant to knowledge, skills or abilities measured by the assessment. For example, consideration should be given to whether a student will spend disproportionate time and effort interpreting material; be exposed to distracting and needless imagery; be unfamiliar with the assumed context, customs or colloquialism of a task.

1.43 The MSOR community supports providers' accessible course development through a variety of resources, for example, the LMS website Mathematics and Accessibility.

Sustainability

1.44 MSOR has a vital role to play in achieving the UN's Sustainable Development Goals, underpinning many technological, scientific and digital developments which have potential to improve health, drive economic growth, transform societies and enhance our environment. For example, mathematical models inform forecasts of climate change, analysis of health data informs public health provision and algorithms help users optimally navigate transport networks. Policies which encourage sustainable development and reduce inequalities can be developed and analysed based on mathematical models and data analysis. MSOR degrees are themselves a driver of social mobility with many graduates from a range of socio-economic backgrounds earning high incomes.

1.45 MSOR is such a versatile subject that many of the 17 UN Sustainable Development Goals could be discussed in the context of MSOR degrees. MSOR is often taught using real-world examples or in the context of applications in other disciplines. Through these examples and applications graduates can appreciate how MSOR can help society to achieve the UN's sustainable development goals. Further, the skills developed through MSOR degrees, such as critical thinking and problem-solving, are useful in understanding, analysing and resolving issues in complex systems such as ecosystems, societies and networks which are impacted by unsustainable development.

1.46 The [Education for Sustainable Development Guidance] produced by QAA and Advance HE outlines pedagogic approaches for implementation in UK higher education institutions. In the context of MSOR degrees these might include the following.

- Projects or dissertations where the focus is on modelling or analysing a problem connected to sustainability. The project or dissertation may be interdisciplinary in nature and involve working with students or supervisors in other fields. Topics could include, for example, modelling energy needs or measuring ecological biodiversity.
- Specific modules which focus directly on the use of MSOR in addressing a specific sustainability issue. For example, mathematical medicine or environmental modelling could be the basis of a module.
- Case studies which illustrate the applicability of an MSOR method or technique to a sustainability issue; or case studies which have motivated new MSOR research.
- MSOR problems where the motivation or context of the question is a sustainability issue.
- Discussion of inequalities, perhaps in the context of professional ethics, protected characteristics and/or equality, diversity and inclusion.
- Consideration of ethical issues and unintended consequences of MSOR, such as the environmental impact of high-performance computing or the use of pure areas such as graph theory in controversial social media practices.

1.47 The following are examples of how MSOR methods may be linked to sustainability issues.

- Pollution levels connected with transport could be reduced by applying fluid dynamics to improve aerodynamic efficiency or optimisation algorithms to reduce delays in networks.
- Population dynamics can be modelled using ordinary differential equations in the context of species growth and decline, using SIR (susceptible, infected, removed) models in the context of epidemics or using network theory applied to ecosystems.
- Automated diagnosis based on medical images may be achieved using classification algorithms and low rank approximations of images using matrix factorisation.
- There are various mathematical models for climate forecasting, such as those based on Navier-Stokes equations, and extreme value theory to estimate the risks of weather events.
- Machine learning, artificial intelligence and data science have many applications in sustainability in the contexts of, for example, energy, resource management, biodiversity, crop yields and climate.
- Pure mathematics is applied in cryptography and blockchain techniques, which have significant environmental consequences.

1.48 Sustainable development may be revisited multiple times in the curriculum to reinforce the connections between different areas of MSOR and economic, social and environmental issues. Education for sustainable development is an emerging field within MSOR and providers are encouraged to innovate and evaluate pedagogical developments in this area.

Enterprise and entrepreneurship education

1.49 In general, as articulated in QAA's guidance, enterprise and entrepreneurship education supports behaviours, attributes and competencies that are likely to have a significant impact on the individual student in terms of successful careers. It prepares students for changing environments and provides enhanced impact through placements and activities that build links between academic institutions and external organisations.

1.50 Preparing students of MSOR to move successfully from education into employment is an essential part of a degree course. While there is a personal responsibility for students to take ownership of their career development, and other departments within institutions will provide support, development of employability skills should be embedded into MSOR courses, including through enterprise and entrepreneurship education.

1.51 Enterprise is defined here as the generation and application of ideas, which are set within practical situations during a project or undertaking. This is a generic concept involving creativity and problem-solving that can be situated in an MSOR context and applies across all areas of professional life. MSOR courses are particularly well suited to developing this, with a generalised focus that will prepare students for a range of possible future careers.

1.52 Entrepreneurship is defined as the application of enterprise behaviours, attributes and competencies into the creation of cultural, social or economic value. This can, but does not exclusively, lead to venture creation.

1.53 Enterprise and entrepreneurship education is defined here as the process of developing students in a manner that provides them with an enhanced capacity to generate and evaluate ideas, and to broaden their behaviours, attributes and competencies to implement them.

1.54 A key part of any MSOR degree is its focus on problem-solving, which is one of the core competencies associated with enterprise and entrepreneurship education, along with problem identification, creativity and strategic thinking. Other relevant behaviours, attributes and competencies developed through many MSOR modules are adaptability, curiosity, determination and resilience.

1.55 Modules such as a final dissertation or project can foster a student's ability to reflect, take responsibility and take risks in a supervised environment. Modules enabling students to work in groups can provide opportunities for developing and improving skills in influencing, leadership, negotiation and communication. In advance of any group work, students should be supported in developing an appreciation of the differences between working individually and working with others, group management and general good group working behaviours. An ability to work with and interrogate large data sets is key to successful entrepreneurship and enterprise and a module that provides this opportunity will enable students to develop many of the competencies mentioned above.

1.56 A problem-based learning approach in taught modules can help students to appreciate the complex nature of real-world problems. This approach can also help students to develop resilience and encourage them to be adaptable in their approach to problem-solving. These skills are sought after by employers.

1.57 Project work for simulated or real clients can be especially effective in embedding entrepreneurship and enterprise skills in the curriculum and can be a satisfying alternative to placements which provides students with opportunities to develop similar skills. When possible, collaborating with employers on the design of student projects can be a mutually beneficial endeavour which helps to ensure that academic institutions are setting projects that are current, relevant and authentic. Consulting with employers on suitable outputs can lead to a range of possible authentic assessment strategies, with posters, executive summaries and client reports being just some of the possible assessed outputs.

1.58 As graduates can find rewarding employment in many different areas, it is useful for academic institutions to organise regular, targeted events for students where they can hear from and network with employers and alumni.

1.59 During their course, students may have the opportunity to apply for a wide variety of placement opportunities. Students reap many benefits from placements, especially in terms of skills development. Placement opportunities can also help students to develop entrepreneurial skills such as an awareness and appreciation of business, cultural or societal considerations and priorities. By undertaking placements, students are able to see how MSOR work can make a significant contribution to understanding and tackling complex problems in an organisation.

1.60 In order for graduates to build on these skills in their future career or further studies they need to be able to reflect, understand and articulate how their degree has given them the opportunities to develop many areas of entrepreneurship and enterprise as well as general employability skills.

2 Distinctive features of MSOR courses

Design

2.1 Some courses are concerned more with the underlying theory of the subject and the way in which this establishes general propositions leading to methods and techniques, which can then be applied elsewhere. Other courses are more concerned with understanding MSOR results, methods and techniques and their application in MSOR and beyond. For convenience, these different 'styles' will be referred to, respectively, as 'theory-based courses' and 'practice-based courses' (see paragraph 1.10).

2.2 While there are a few courses that are entirely theory or practice-based, most have elements of both approaches and there is a complete spectrum of courses covering the range between the two extremes. It is possible for courses with the same title to have very different emphases; it is the curriculum of a course, rather than its title, that makes clear its position within the spectrum. These different emphases are all equally valuable.

2.3 Possible paths to undergraduate degree study include academic routes, apprenticeships and vocational routes, with some providers offering routes for students with relevant access qualifications. The majority of undergraduate MSOR courses require an A Level or equivalent in a relevant MSOR subject area, for example A Level Mathematics. Some also require A Level Further Mathematics or have other admissions requirements, such as STEP (Sixth Term Examination Paper). Some courses offer foundation years which are designed to provide preparatory work for Level 4 study in the subject. Equally, some applicants may undertake international foundation year courses allied with MSOR degree providers. Such foundation courses may be studied as standalone courses or may be integrated into the degree to enable direct progression to Level 4 of the degree.

2.4 Possible routes to postgraduate taught degrees also vary depending on the nature of provision. The broad discipline area of MSOR encompasses several types of master’s degrees. These include conversion master’s provision and specialist master’s degrees. Conversion provision is often in the areas of data science and statistics, where students may come from a variety of backgrounds with varying levels of MSOR content. More specialist provision is often in areas such as pure mathematics, where students need the mathematical rigour obtained from an appropriate undergraduate qualification.

2.5 Undergraduates in MSOR may choose to study a standard bachelor’s degree with honours (FHEQ Level 6; FQHEIS Level 10) or an integrated master’s degree with honours (FHEQ Level 7; FQHEIS Level 11). These each have distinct learning outcomes to reflect the level of the award. Bachelor’s degrees should provide students with the subject-specific knowledge, understanding and skills, as well as the wider transferable skills and attributes that prepare graduates for a wide range of careers in many sectors.

2.6 Integrated master's degree with honours courses, such as MMath, encompass both bachelor's degrees with honours and master's degree outcomes. An integrated master's degree is awarded after an extended course which allows students to study an MSOR subject to a greater breadth and/or depth than is possible on a bachelor's course, and to extend the opportunities to develop specialist knowledge, advanced skills and undertake more extensive project work. These master's degrees thus provide a coherent, integrated opportunity to develop a deeper and/or wider level of knowledge, understanding and experience, sufficient to prepare students for a professional career in MSOR. Standalone master's degrees in MSOR, such as MSc and MRes, are self-contained courses, normally involving the equivalent of one or two years of full-time postgraduate study in a specialist area.

2.7 In addition to single honours courses in MSOR subjects, many joint honours courses studying MSOR in combination with another distinct discipline are available. In joint honours courses, undergraduates will study core elements of the specific and generic skills for each subject, and will add others according to the modules on offer within the degree. Additionally, students may explore the overlap between their two subject areas, creating opportunities for interdisciplinary study. In interdisciplinary degrees, such as data science, students will study a set of core knowledge across the subject areas while potentially specialising in one or more of the component disciplines.

2.8 Some MSOR courses will enable students to learn outside the formal academic environment. This may be through placements or internships in industry or educational settings, for example, within schools or colleges through initiatives such as undergraduate ambassador schemes, or to study at an international university. Such placements may last for a few weeks, a term, a semester or an entire year and can also be arranged on a part-time basis. They typically take place after completion of the equivalent of at least two full years of study.

2.9 Both bachelor's and integrated master's degrees may include such periods of study, and this may or may not extend the period of the degree, depending on the expected learning undertaken during that period. Credit awarded during such study also varies according to the learning and assessment workload during the experience. Credit-bearing placements (see paragraph 2.27 in the Partnership section) should, however, be integrated within the programme of study, so that students can relate their experience to, and use the skills that they have developed in, their academic study. Many providers also offer or facilitate non-credit-bearing industrial and research placement experiences during vacations to enhance student experience and development.

2.10 Where a provider offers several MSOR courses, these may be based around a common core of shared compulsory modules, especially in the early years, with options in later years that allow students to specialise. This approach enables both flexibility and efficiency of delivery and may even allow students to defer selection of the award title until later years, by retaining the option to transfer between cognate courses (see paragraph 2.17 on Flexibility).

2.11 The academic component of integrated degree apprenticeships in MSOR should follow the guidelines in this Subject Benchmark Statement. Degree apprenticeships in higher education are covered explicitly in the [Characteristics Statement for Higher Education in Apprenticeships] which describes the general characteristics and distinctive features of higher and degree apprenticeships in the UK.

Progression

2.12 Over the course of a bachelor's degree with honours (FHEQ Level 6; FQHEIS Level 10) or an integrated master's degree (FHEQ Level 7; FQHEIS Level 11), an MSOR student will progress from one level of study to the next, in line with the regulations and processes for each institution. However, it is expected that each level would see the attainment of knowledge, expertise and experience that build towards the final achievement of meeting at least the threshold-level subject-specific and generic skills listed in this Statement. This would include successful completion and the award of credit for the full range of learning and assessment. Upon graduation from an undergraduate degree, it would be expected that a student who has achieved a second-class degree or higher would be capable of, and equipped for, undertaking postgraduate study in MSOR or an associated discipline. Entry requirements to postgraduate courses are, however, determined by individual providers and may require specified levels of achievement at undergraduate level.

2.13 Undergraduates studying joint honours or interdisciplinary courses will achieve core elements of the specific and generic skills for each subject (see paragraph 2.7).

2.14 Integrated master's degrees (FHEQ Level 7; FQHEIS Level 11) typically require the equivalent of an additional full-time year of study when compared to a bachelor's degree.

2.15 In undergraduate honours degree courses, students may exit earlier and be eligible for a Certificate of Higher Education, a Diploma of Higher Education, or a pass or ordinary degree depending upon the levels of study and credit completed to a satisfactory standard. In Scotland, bachelor's degrees with honours are typically designed to include four years of study, which relates to the structure of Scottish primary and secondary education. For students following part-time routes, their study time would be the equivalent of the three or four-year degree.

Flexibility

2.16 The progressive acquisition of knowledge and skills within the subject area also enables flexibility between courses, both within and between institutions.

2.17 As has been set out in paragraph 2.10 in the Design section, MSOR courses designed around a common core of shared compulsory modules in the early years, followed by specialisation in later years, facilitate both flexibility and efficiency of delivery. They can also enable students to defer selection of their award title until later years and can make the transfer between cognate courses more straightforward.

2.18 MSOR students should also have flexibility to select some optional modules in alignment with their interests, and therefore appropriate guidance on option choices should be made available to the students.

2.19 The cumulative nature of the subject limits the flexibility that can be designed into some MSOR courses that require certain prerequisites for certain modules. This will also limit the flexibility of the credit transfer process as, in such cases, students will require a sufficient number of specific credits that cover the prerequisites in order to transfer onto some MSOR courses.

2.20 In England, some higher education providers use a flexible approach to the staging of modules in order to offer courses where learners spend part of their final year in a school in order to gain Qualified Teacher Status (QTS) and a full honours MSOR degree without extending the duration of the degree course.

2.21 Flexible educational approaches enable learners to adapt their education to their situational and contextual individual needs and constraints, and may also play a key role in increasing access to further and higher education and social mobility. For example, such approaches could provide scope for learners to select educational opportunities that are better suited to their current level of proficiency and interests (see paragraphs 1.22 and 1.40).

2.22 To this end, it is beneficial for higher education providers to have:

- flexible approaches to assessment tasks that enable learners to demonstrate different competencies
- flexible approaches to awarding and transferring credits, for example the integration of alternative recognition of learning (such as prior experiential learning and micro-credentials) with traditional modular credits
- flexible approaches to recruitment processes that recognise prior learning and/or work experience in the field of MSOR.

2.23 In addition, across the sector, it is beneficial to have:

- flexible delivery modes, including, but not limited to, campus-based, distance learning, block release and hybrid (campus-based and distance learning)
- flexible study patterns in terms of intensity of study and start dates.

2.24 MSOR courses should be sufficiently flexible to be able to respond to and anticipate change, in the advancement of the subject and its interface with other disciplines, as well as in the needs of its graduates and their employers. They should also take account of learners' needs and make appropriate reasonable adjustments, as required.

Partnership

2.25 Partnerships may be academic, as a collaboration with other educational organisations, or may be professional, as a collaboration between providers and industry.

2.26 An academic partnership may be between providers and other UK or international educational institutions. These may include partnership programmes that give advanced standing to students with prior study abroad, for admission into year two or three of existing UK degree courses.

2.27 Partnerships between providers and industry can involve employers or professional organisations and may include, but are not limited to, placements and work-related projects. These can be offered, organised and advertised by external organisations, but can also be developed as a collaborative effort between higher education providers and employers. They may be part of the formal curriculum, be awarded credit and the employer might contribute to assessing the students' work.

2.28 Placements may include working for an employer for a short time each week over the length of a module or in a block in or outside term time. Employers include private and public sector organisations as well as charities and other voluntary groups. Organisations such as schools and colleges can provide placements for MSOR students analysing and presenting student data or applying MSOR in areas such as timetabling. Work can be focused on a specific project or include a variety of tasks as directed by the employer.

2.29 Many employers are keen to take on MSOR students for a year-long sandwich placement between years of study. MSOR students are in demand especially due to their potential skills around handling data and programming as well as their knowledge of specific MSOR techniques.

2.30 Students gain numerous benefits from working with employers on placements or projects (see paragraph 1.59). Work-related projects may involve employers posing problems for students to work on which have arisen or are inspired by the business of the employer. Some of these are detailed in paragraph 1.57. Any partnership with a potential employer or professional organisation can lead to input into curriculum design or other activities that enhance the curriculum as well as students' employability and graduate outcomes. Some providers offer degree apprenticeships, for example in data science, in conjunction with employers. Providers should develop processes for oversight to ensure that partners involved in degree delivery meet all relevant safeguards, quality standards and expectations.

Monitoring and review

2.31 Degree-awarding bodies, and their collaborative partnerships, routinely collect and analyse information and undertake periodic course review according to their own needs.

They draw on a range of external reference points, including this Subject Benchmark Statement, to ensure that their provision aligns with sector norms. Monitoring and evaluation is a periodic assessment of a course, conducted internally or by external independent evaluators. Evaluation uses information from both current and historic monitoring to develop an understanding of student achievement or inform future course planning.

2.32 Externality is an essential component of the quality assurance system in the UK. Providers will use external reviewers as part of periodic review to gain an external perspective on any proposed changes and ensure threshold standards are achieved and content is appropriate for the subject.

2.33 The external examination system currently in use across the UK higher education sector also helps to ensure consistency in the way academic standards are secured by degree-awarding bodies. Typically, external examiners will be asked to comment on the types, principles and purposes of assessments being offered to students. They will consider the types of modules on offer to students, the outcomes of a cohort and how these compare to similar provision offered within other UK higher education providers. External examiners are asked to produce a report each year and make recommendations for changes to modules and assessments (where appropriate). Subject Benchmark Statements, such as this one, can play an important role in supporting external examiners in advising on whether threshold standards are being met in a specific subject area.

2.34 MSOR courses can achieve accreditation from various professional bodies, each of which have specific requirements to award the accreditation (see paragraph 1.20).

3 Content, structure and delivery

Content

Introduction

3.1 As discussed in previous sections, the subject area of MSOR covered by this Statement is very broad. Therefore, the knowledge and skills that may be expected of graduates in the area are correspondingly wide-ranging.

3.2 Courses develop graduates who have knowledge and skills that are specific to areas within MSOR. In this Subject Benchmark Statement, such knowledge and skills are referred to as subject-specific. At the higher levels of study, this knowledge and these skills naturally vary between graduates because of the different areas of the subject(s) that different students pursue. This diversity, which is a natural feature of the MSOR subject area, is to be welcomed, and must not be restricted in any way. Furthermore, it is dynamic and evolving, as courses develop to encompass new areas of study. It is, however, possible to discern subject-specific knowledge and skills that are demonstrated by all MSOR graduates.

3.3 Although most of the foundations of knowledge and skills in MSOR are generally laid in the earlier parts of courses, this is not exclusively the case. Equally, the earlier stages are not necessarily exclusively concerned with laying foundations; in many courses it may be entirely proper for more advanced work or for work on applications to start at an early stage, provided always that any necessary prerequisite knowledge is in place.

3.4 No attempt has been made to construct a comprehensive listing of subject-specific knowledge for all courses covered by this Statement. Such a listing would be far too prescriptive, may well force unnecessary and undesirable modifications in some existing courses, and would confer no positive benefits.

3.5 Courses in the MSOR area will also produce graduates who have highly developed skills of a more general kind. Obvious examples are that they should be highly numerate and that most graduates are competent with applications in computing and modelling.

3.6 MSOR subjects are central to data-related analysis. For example, data science has emerged as an interdisciplinary field combining MSOR subjects, computer science and others. It is expected that many graduates will wish to pursue careers in data-related activities and MSOR courses will play a leading role in supplying these graduates.

Subject-specific knowledge and understanding

General principles

3.7 Courses develop graduates who have knowledge and understanding of methods and techniques appropriate to their main field of study, and from a range of other areas of MSOR. In addition, most graduates have met at least one major area of application of their subject. Different types and levels of understanding are developed in the graduates according to the focus of the course.

Methods and techniques

3.8 Common ground for all courses includes calculus, linear algebra and probability/ statistics. All courses cover methods and techniques that pertain to a range of areas of MSOR, developed in depth according to their own requirements. As examples, graduates from courses in operational research may have considerable knowledge of constrained optimisation and its application to allocating scarce resources, or of modelling different decision-making processes; whereas graduates from courses concentrating on applications of mathematics in physics or engineering may have correspondingly deep knowledge of methods for working with differential equations. These examples have been deliberately chosen as being fairly far apart in the spectrum of MSOR courses, but it is to be emphasised that the methods and techniques covered in them are not mutually exclusive.

Areas of Mathematics, Statistics and Operational Research

3.9 Courses develop graduates who will have knowledge of a range of areas of MSOR with the level of understanding of derivation of results and their application commensurate with the course's placement on the spectrum between practice-based and theory-based.

3.10 Mathematics courses develop graduates who have in-depth knowledge of concepts, methods and techniques in a range of mathematical topics. Examples may include algebra, analysis, geometry, number theory, differential equations, continuum mechanics and mathematical physics, but there are many others. Graduates may also have understanding of mathematical models and how and when they can be applied.

3.11 Statistics courses develop graduates who have knowledge of core areas of mathematics and a range of major areas in statistics. Examples may include exploratory data analysis, inference, likelihood, linear models, stochastic processes, time series and data analytics. Graduates are able to use a statistical package for data analysis.

3.12 Operational research courses develop graduates who have experience of a wide range of applications. These may include examples from fields such as healthcare, transportation, logistics, strategic planning, manufacturing and retail distribution. It is often the case that specialised modules in these areas are available, sometimes being taught by staff from the respective subject departments.

Mathematical thinking and logical processes

3.13 Courses develop graduates who have an understanding of the importance of assumptions and a recognition of where they are used and of possible consequences of their violation. This includes an appreciation of the distinction between the roles of validity of assumptions and validity of arguments. This may also take account of, where appropriate, the ethical consequences of assumptions.

3.14 Courses develop graduates who also appreciate the power of generalisation and abstraction in developing mathematical theories or methods to use in problem-solving. Theory-based courses may tend to emphasise the role of logical mathematical argument and deductive reasoning, often including formal processes of mathematical proof; practice-based courses may tend to emphasise understanding and use of structured mathematical or analytical approaches to problem-solving. In both cases, problems often include unfamiliar small or large-scale tasks that may require the independent evaluation, analysis and selection of appropriate strategies and techniques applied to create a complete solution. Resilience and reflection over longer timescales may be required to learn from approaches that are not successful to develop the final successful approach.

3.15 Knowledge and understanding under this heading inform and underpin many other activities that may appear in various courses, such as axiomatic approaches to advanced pure mathematics or the general role of modelling.

Numerical computation

3.16 Courses develop graduates who have knowledge and understanding, at the level required for their courses, of some processes and pitfalls of numerical approximation.

3.17 Courses develop graduates who have some knowledge and understanding of programming in service of computation. With computation regularly used in the workplace and familiarity with software packages an essential part of data analysis, graduates will have familiarity with at least one programming language and direct experience of specialist software for the problems being addressed and, when feasible, knowledge of the nature of the algorithms on which it is based.

Modelling

3.18 Mathematical and statistical modelling involve abstract representations of processes. All graduates are expected to have some knowledge and understanding of this activity. Generally, the problems come from at least one application area, but they may also come from other areas within MSOR.

3.19 Courses develop graduates who have knowledge and understanding of a range of modelling techniques and their conditions and limitations, and of the need to validate and revise models. Graduates also know how to use models to analyse and, as far as possible, solve the underlying problem or to consider a range of scenarios resulting from modifications to it, as well as how to interpret and communicate the results of these analyses.

Related areas

3.20 Degrees may include content closely related to MSOR subjects and their professional practice, for example, history of mathematics, mathematics education and career development skills. Such content can increase motivation and better prepare students for appropriate employment.

Depth of study

3.21 Courses develop graduates who have knowledge and understanding developed to higher levels in particular areas. The higher-level content of courses is sometimes reflected in the title of the course. For example, graduates from courses with titles involving statistics have substantial knowledge and understanding of the essential theory of statistical inference and of many applications of statistics. Courses with titles such as mathematics may range quite widely over several branches of the subject, but, nevertheless, graduates from such courses have treated some topics in depth. Integrated master's courses generally include aspects of mathematics, statistics and operational research in greater depth and/or breadth than bachelor's courses.

Subject-specific skills

3.22 MSOR graduates have subject-specific skills developed in the context of a very broad range of activities. These skills have been developed to a sufficiently high level to be used after graduating, whether it be in the solution of new problems arising in professional work or in further study, including interdisciplinary work involving MSOR.

3.23 Many of the subject-specific skills to be expected of all MSOR graduates are directly related to the fundamental nature of MSOR as a problem-based subject area, whether the problems arise within MSOR itself or come from distinct application areas.

3.24 Graduates from courses focused on particular branches of MSOR have other subject-specific skills that are relevant to those particular branches. An exhaustive list of such skills is not helpful, but, as examples, graduates from:

- **Mathematics courses** may have skills relating particularly to logical argument and solving problems in generality, and facility with abstraction, including the rigorous development of formal theories. In addition, they may have skills relating particularly to formulating real-world problems in mathematical terms, solving the resulting equations analytically or numerically, and giving contextual interpretations of the solutions
- **Statistics courses** have skills relating particularly to the design and conduct of experimental and observational studies and the analysis of data resulting from them
- **Operational research courses** have skills relating particularly to the formulation of complex problems of optimisation and the interpretation of the solutions in the original contexts of the problems.

General skills

3.25 MSOR courses develop graduates who acquire many general skills honed by their experiences of studying MSOR subjects. All these are essentially problem-solving subjects, whether the problems arise within MSOR itself or come from areas of application. Therefore, the graduates' experiences are embedded in a general ethos of numeracy and of analytical approaches to problem-solving. In addition, an important part of most MSOR courses is to take theoretical knowledge gained in one area and apply it elsewhere. The field of application is often a significant topic of study in its own right, but the crucial aspect of the process is the cultivation of the general skill of transferring expertise from one context to another.

3.26 A number of general skills are to be expected of all MSOR graduates, although these are likely to be developed to different extents in different courses. These lead to competencies that enhance the general employability of MSOR graduates and include:

- study skills, particularly including the ability to learn independently
- the ability to work independently with patience and persistence, pursuing the solution of a problem to its conclusion
- efficient management of processes and deadlines
- adaptability, in particular displaying readiness to address new problems from new areas
- the ability to transfer knowledge from one context to another, to assess problems logically and to approach them analytically
- highly developed skills of numeracy, including being thoroughly comfortable with numerate concepts and arguments in all stages of work
- the ability to apply specialist software appropriately
- information technology (IT) skills and the ability to obtain information from a variety of sources, always taking care that these sources are referred to appropriately
- working in collaboration and building on the work of others
- communication skills, generally including verbal and non-verbal, and to communicate results clearly in various ways (for example, reports, presentations, posters)
- knowledge of ethical issues, including the need for sensitivity in handling data of a personal nature and the impact of MSOR in its application.

3.27 In summary, MSOR courses develop graduate attributes which include an enhancement of many general skills, such as numeracy, IT skills, critical understanding and assessment of complex problems, and the ability to identify and analyse problems leading to formulation of solutions, as well as subject-specific skills such as mathematical modelling, data analysis and numerical methods.

Teaching and learning

3.28 A variety of teaching and learning methods are in use in MSOR. These may differ markedly depending on, for example, the style of the course, the subject matter, the level and progression route of the learners and the resources which are available. Providers should carefully choose their approach, having evaluated a wide range of available pedagogies. It is anticipated that multiple approaches will be necessary, even within a single module, but consideration should also be given to maintaining a coherent experience for the student over the duration of the course.

3.29 Students within MSOR are likely to meet a range of learning activities, including, but not limited to, lectures, small group tutorials and computer laboratories. Students benefit from seeing arguments developed by tutors in 'real time' and it is common for tutors to present extended arguments in classes, particularly in lectures. Some associated implications for physical spaces are detailed in paragraph 3.37. Alternatives to lecture-based teaching in MSOR courses include active learning and flipped classroom approaches, where the primary focus is on student engagement in the activity through discussion, consolidation, or extension activities.

3.30 With any approach, providing appropriate learning materials in advance increases the accessibility of a live class and facilitates preparation work, enabling all students to engage more fully in the session. Regular work on practice exercises is important in many MSOR courses as this helps students to achieve a good understanding of core techniques and methods. In addition to setting practice exercises, it is useful to provide students with opportunities to consider unseen or open-ended problems, and to discuss and critically analyse MSOR discipline content in classes where possible. These activities can help to highlight important aspects of the subject area to learners and encourage the development of graduate skills and attributes.

3.31 A consequence of the cumulative nature of MSOR is that, to a greater extent than for many subjects, topics may need to be revisited at different stages of the course. This helps to reinforce a deeper learning of the material. For example, students may need to revisit topics at the start of a teaching session or at the start of the module if it is later to become an ingredient for more advanced work. This is particularly the case if something familiar is expressed in a different, but mathematically equivalent, way or if a different notation for something familiar is introduced.

3.32 The practice and repetition required for MSOR lends itself to collaborative learning. It is common across a cohort to find a range of strengths and weaknesses, and students vary in the speed in which they gain proficiency in different concepts. Therefore, peer-based learning enriches an individual's learning experience, providing an important and valuable resource. Peer-based learning is facilitated through certain teaching formats, such as workshops, tutorials and seminars. Peer-based learning also exists outside of scheduled teaching activities and can be student-led, including, for example, group study sessions and student society events. Providers should ensure access to appropriate formal and informal learning spaces for students to facilitate group activities. Providers should promote and encourage peer-based learning in a way that follows good academic practice. As such, students develop understanding of how to learn from others without compromising academic integrity.

3.33 A consequence of the need to practice developing skills and techniques, the revisiting of topics at multiple points in the course and the fact that the discipline builds on content learned at school is that some students may need additional support as they come to understand MSOR content. Providers may need to resource availability of tutors for individual help outside of scheduled classes and promote the effective use of such sessions. Some providers additionally operate a formal peer support scheme, perhaps involving students from later stages of the course as tutors. Some providers will offer a mathematics and statistics support service either through the same department that is offering the MSOR course or otherwise. This is an MSOR-specific facility offered to students of multiple disciplines which provides extra assistance in addition to course-based teaching and learning, including via workshops, drop-in sessions, and one-to-one appointments, that may be suitable for MSOR students seeking additional help.

3.34 Providers should acknowledge their responsibility, shared with their students, for developing their students into independent scholars and practitioners of their disciplines; this includes, but is not limited to, recognising the need to provide appropriate scaffolding, confidence building and development opportunities to enable students to become effective independent learners.

3.35 A characteristic of MSOR is that significant quantities of domain knowledge and practice of associated skills are required before a student is able to independently and meaningfully engage with solving more advanced problems, especially in more abstract areas. Providers should prioritise equipping students with the discipline-based skills to enable them to independently learn MSOR content and recognise that in doing so their provision is enriched by the nurtured potential of their students.

3.36 Providers should assist students in developing effective study skills so that they can actively engage with teaching and resources. Tutors should model and explain the value of spending time on tasks which do not easily fit within formal timetabled hours. These tasks may include reading ahead or around the topic, diligently checking mathematical arguments, or independent practice of skills, including problem-solving and programming. Scholarly behaviours, such as playing and experimenting with ideas, challenging the necessity of particular assumptions, or attempting to solve a question, even if the solution is incomplete or later found to contain errors, should be encouraged. Students will need time to consolidate their understanding and develop MSOR discipline skills, perhaps through spaced repetition, and providers should timetable teaching hours and schedule assessments to accommodate this need.

3.37 A range of different teaching spaces are typically required for MSOR classes. These include, but are not limited to, lecture theatres with suitable boards and projection facilities, computer laboratories, and rooms with adaptable table and chair configurations to facilitate peer discussions or group work. Where a board-based approach is adopted, considerable board space may be required for the display of the teaching material. It is sometimes necessary to display distinct material simultaneously, such as digital output from specialist software at the same time as arguments being written on boards. Teaching spaces may require appropriate facilities for this as well as equipment to allow useful recordings of relevant sessions to be made.

3.38 Effective use of technology to facilitate and enhance teaching and learning is widespread and expected in MSOR courses. Examples include specialised software that assesses mathematical input and can offer immediate personalised feedback to students, and interactive web applications that allow students to explore MSOR concepts in depth, typically including visual interfaces.

3.39 Computational work is found across MSOR. In teaching and learning, computing for MSOR may be concentrated within modules or embedded across a course, depending on the emphasis of the course. Students develop skills in computational thinking, should encounter at least one programming language, and may use specialist software to carry out technical MSOR work. Examples include:

- computer algebra systems
- specialised software for particular areas of algebra
- programs for numerical analysis
- statistical packages for data analysis and model building
- mathematical programming software for formulating and solving operational research problems.

This is in addition to the use of standard spreadsheets, graphics systems and specialised systems for mathematical typesetting.

3.40 A wide range of learning materials are used to support the delivery of MSOR courses. These include:

- self-contained lecture notes
- lecture slides
- textbooks
- journals
- video recordings
- problem sheets and model solutions
- diagnostic and formative quizzes
- computer code
- interactive simulations
- forums
- other online digital material.

3.41 Typically, MSOR staff develop bespoke resources, such as lecture notes and practice questions, for each module rather than following a specific reference. This is a consequence of modules being specifically developed to fit within the overall course design of the provider, including the prerequisite structure, and a desire for greater flexibility over teaching resources. Consequently, MSOR staff need to routinely update materials and practices in response to both developments in domain knowledge and pedagogy.

3.42 As in all subjects, MSOR teaching is continually evolving, with providers and individual members of staff responding to changing circumstances and changes to wider educational practice. Innovation in learning, aimed at improvement, is to be welcomed, with the outcomes critically evaluated and disseminated. Providers should support new MSOR staff to take part in relevant training appropriate to their discipline. Where postgraduate tutors assist staff in the delivery of the programmes, the provider should ensure that postgraduate tutors receive sufficient discipline-relevant training to best support them in this role. All MSOR staff have a responsibility to engage in continuing professional development throughout their career. To support the ongoing enhancement of MSOR education there are many continuing professional development opportunities for staff run by the MSOR community.

3.43 Providers should engage with quality assurance processes to maintain and enhance the quality of their provision; some of these are locally determined, others are statutory in nature and linked to the retention of degree awarding powers, and others are linked to accreditation by external bodies.

Assessment

3.44 Carefully designed assessments enable students to demonstrate their achievement of the intended learning outcomes. Assessment in MSOR ultimately aims to determine whether students meet the standards outlined in section 4, including establishing the student's:

- knowledge and understanding of a corpus of well-established material
- operational understanding of the conventions of the discipline, for example, the norms of mathematical proof or statistical inference
- ability to apply knowledge, work with abstract models, solve problems and to reason rigorously
- ability to communicate clearly and accurately in a variety of verbal and/or non-verbal formats, including appropriate use of mathematical symbols.

3.45 Summative assessment could also establish the student's abilities beyond understanding MSOR subject knowledge, such as to:

- pursue substantial independent projects and write reports
- use existing software and write code
- interpret, evaluate and critique the work of others
- work effectively within a team.

3.46 Assessment can facilitate students' learning by providing constructive feedback on their performance relative to the intended learning outcomes. Such formative feedback helps students understand how performance could be improved, with subsequent opportunities to apply what has been learned. This feedback can take a variety of forms: it may be individual or collective, and written or oral. Informal feedback may be provided in addition to formal recorded feedback.

3.47 MSOR has important general characteristics which pervade the culture of the discipline and can be taken to apply to all specific courses as intended learning outcomes. For example, minor symbolic differences can radically change the intended meaning, and in applications inaccuracy can have very serious consequences. Similarly, spatial awareness; appreciation of patterns; agility in complex logical reasoning; writing proof; and clarity of expression are all important. Some assessments, or some components of assessments, will seek to establish whether students can demonstrate these intended general learning outcomes. Assessment design should consider equality, diversity and inclusion issues as outlined in paragraph 1.27.

3.48 A wide variety of methods may be used for assessment according to context and purpose, recognising that learners may exhibit different aptitudes in different forms of assessment. Automatically marked online assessment has widespread application, particularly for formative work. All assessments, including traditional invigilated examinations, should be designed taking into consideration both matters of accessibility and factors around equality, diversity and inclusion.

3.49 Students benefit most where time has been invested in developing skills which match the format of assessments. For example, where presentations form part of a summative assessment, students benefit from opportunities to understand what is required, such as in using visual aids and developing practice in presenting.

3.50 Setting and marking assessment tasks requires a great deal of professional judgement. While some assessment seeks to establish objective properties, for example, 'correctness', marking is not always, as is often thought, entirely deterministic. Examiners are often required to judge quality, for example, when evaluating a mathematical model, assessing the discussion of data, or awarding grades to solutions that are flawed but not wholly incorrect and to different presentations of correct material.

3.51 Some areas of MSOR require particular assessment types to ensure the validity and integrity of the assessment. Controlled conditions, such as those provided by traditional invigilated examinations, are often essential. For example, the nature of the subject means that sometimes students can be expected to provide an answer which is very close to a model answer. Such controlled conditions also help ensure that only work produced by students themselves is being assessed, for example by following the principles in QAA's guidance Contracting to Cheat in Higher Education.

3.52 In MSOR, assessment marks can span the entire percentile range. It is possible for an MSOR student to produce a solution that is entirely correct and therefore warrants being awarded full marks. It is an inherent feature of the subject that the performance of an individual student may vary significantly between modules and a student's marks on some modules may not be aligned with their overall performance. Students towards the lower end of the performance range may fail some modules while still meeting the overall learning outcomes of the course. Towards the upper end of the performance range, students may still fail individual modules. MSOR is often best served by allowing examiners to judge the overall performance of a student against the learning outcomes for the whole course by using averaging or preponderance systems which take an overall view of a student's achievements. Consequently, some aspects of provider-wide assessment regulations might be less applicable in MSOR than other subjects and may need to be adapted to take account of the inherent nature of the discipline, otherwise the professional judgement of the examiners (internal and external) could be seriously compromised.

4 Benchmark standards

Introduction

4.1 The subject area covered by the Subject Benchmark Statement for MSOR is very wide, and therefore the standards that may be expected of graduates in the area can only be specified in a fairly general way.

4.2 Benchmark standards for MSOR are defined at threshold levels of competence for a bachelor's degree with honours and for integrated master's and taught postgraduate degrees. It is intended that students meet these standards in an overall sense, not necessarily in respect of each and every criterion listed.

4.3 Aspects of MSOR are integral parts of data science courses and joint courses with many other subjects. This Statement is a reference point in these cases, in so far as it can be applied to relevant parts of courses in which the MSOR content is only a proportion of the whole.

4.4 The vast majority of students will perform significantly better than the minimum threshold standards. Each higher education provider has its own method of determining what appropriate evidence of this achievement will be and should refer to Annex D: Outcome classification descriptions for FHEQ Level 6 and FQHEIS Level 10 degrees. This Annex sets out common descriptions of the four main degree outcome classifications for bachelor's degrees with honours: 1st, 2.1, 2.2 and 3rd. Students graduating at master's level will build on the outcomes expected at Levels 6/10 but focus in greater depth on a specific subject area and achieving outcomes as described at this level in the Frameworks for Higher Education Qualifications.

Academic standards

4.5 A graduate who has reached the bachelor's degree with honours threshold level should be able to demonstrate:

- a reasonable understanding of the basic body of knowledge for the course of study, normally including calculus, linear algebra and probability/statistics, and an ability to comprehend the basic language of the discipline
- a reasonable level of skill in calculation and manipulation within this basic body of knowledge and some capability to solve problems formulated within it
- application of concepts and principles in well-defined contexts, showing judgement in the selection and application of tools and techniques and demonstrating justification of the methods used
- reasoning using logical arguments, including identifying the assumptions made and the conclusions drawn
- a reasonable level of skill in comprehending problems, formulating them mathematically, obtaining solutions by appropriate methods, and drawing valid inferences from these
- an ability to communicate logical arguments, evidence and conclusions reasonably accurately and clearly, including, if appropriate, acknowledging the degree of uncertainty associated with conclusions
- familiarity with at least one programming language and competent use of other appropriate MSOR technology.

4.6 A graduate who has reached the integrated master's degree threshold level should be able to demonstrate:

- a good understanding of the main body of knowledge for the course of study, including some advanced topics
- a good level of skill in calculation and manipulation of the material within this body of knowledge, and be capable of solving advanced problems formulated within it
- application of a range of concepts and principles in loosely defined contexts, showing good judgement in the selection and application of tools and techniques and demonstrating justification of the methods used
- a good level of capability in developing and evaluating logical arguments
- a good level of skill in comprehending problems, formulating them mathematically, obtaining solutions by appropriate methods, and drawing valid inferences from these
- confident and effective communication of logical arguments, evidence and conclusions accurately and clearly, and, if appropriate, acknowledging the degree of uncertainty associated with conclusions
- familiarity with at least one programming language and competent use of other appropriate MSOR technology
- the ability to work competently and independently, to be aware of own strengths and to understand when help is needed
- competence in planning and conducting an advanced project in mathematics, statistics or operational research or a related topic area.

4.7 Taught postgraduate master's degrees will develop some or all of the standards listed above, depending on the precise nature of the course of study, which may be specialised within a subdiscipline or interdisciplinary with another discipline area.

4.8 Standards above the threshold may be demonstrated through:

- the depth of the student's understanding of concepts or techniques
- the breadth of the student's knowledge
- the amount of support and guidance the student requires to undertake an extended task
- the complexity of the problems that the student can solve or model and interpret
- the ability to go beyond applying a given method and construct creative and innovative solutions to unfamiliar problems within the context
- the student's ability to construct and present a reasoned argument or proof and how far the student can progress through it
- the facility with which the student performs calculations or manipulations.

Professional standards

4.9 Alongside the academic standards listed above, students will develop transferable skills and awareness of standards needed to use MSOR in a diverse array of professional settings. These are in addition to general professional behaviours and attributes expected of graduates. Specifically, MSOR graduates should be expected to:

- synthesize information from different branches of the subject
- apply critical thinking and problem-solving skills to problems arising in MSOR and in the context of other disciplines or applications
- work effectively individually or as part of a group to achieve a specific aim
- communicate accurately and clearly verbally and/or non-verbally using a variety of formats; this may include communication to specialist and non-specialist audiences

- possess digital capabilities, including retrieval of information and the assessment of its veracity, the use of generic IT, and the ability to select and employ appropriate digital tools to support mathematical thinking, reasoning and communication
- be aware of the need to work within existing professional and legal frameworks, including data security, EDI and sustainability, each within an MSOR context
- maintain appropriate ethical standards and be aware of the broader implications of their work
- be able to initiate self-learning and monitor/adjust personal programmes of work in light of reflection and feedback received.

4.10 Students at master's level will have had a deeper or broader experience of a subset of the skills listed and greater experience of working independently, with additional opportunities to form novel and creative approaches for tackling non-standard problems.

5 List of references and further resources

QAA (2019) The UK Quality Code for Higher Education
www.qaa.ac.uk/quality-code

QAA and Advance HE (2021) Education for Sustainable Development Guidance
www.advance-he.ac.uk/teaching-and-learning/education-sustainable-development-higher-education

QAA (2018) Enterprise and Entrepreneurship Education: Guidance for UK Higher Education Providers
www.qaa.ac.uk/docs/qaa/about-us/enterprise-and-entrpreneurship-education-2018.pdf

QAA (2019) Characteristics Statement: Higher Education in Apprenticeships
www.qaa.ac.uk//en/quality-code/characteristics-statements/higher-education-in-apprenticeships-characteristics-statement

QAA (2019) Annex D: Outcome classification descriptions for FHEQ Level 6 and FQHEIS Level 10 degrees
www.qaa.ac.uk/docs/qaa/quality-code/annex-d-outcome-classification-descriptions-for-fheq-level-6-and-fqheis-level-10-degrees.pdf

QAA (2022) Contracting to Cheat in Higher Education: How to address Essay Mills and Contract Cheating (3rd Edition)
www.qaa.ac.uk/docs/qaa/guidance/contracting-to-cheat-in-higher-education-third-edition.pdf

Maths Careers Website
www.mathscareers.org.uk

Measuring the Economic Benefits of Mathematical Science Research in the UK.
www.lms.ac.uk/sites/default/files/Report%20EconomicBenefits.pdf

The Era of Mathematics: An Independent Review of Knowledge Exchange in the Mathematical Sciences.
https://webarchive.nationalarchives.gov.uk/ukgwa/20220208115701/http:/epsrc.ukri.org/newsevents/pubs/era-of-maths/

Institute of Mathematics and its Applications
https://ima.org.uk

Royal Statistical Society
https://rss.org.uk

The Operational Research Society
www.theorsociety.com

The Institute and Faculty of Actuaries
https://actuaries.org.uk

London Mathematical Society
www.lms.ac.uk

London Mathematical Society Good Practice Scheme
www.lms.ac.uk/women/good-practice-scheme

Mathematics and Accessibility, London Mathematical Society
https://www.lms.ac.uk/policy/mathematics-and-accessibility

The Council for the Mathematical Sciences
www.cms.ac.uk/wp/2015/10/28/the-mathematical-sciences-people-pipeline/

United Nations Department of Economic and Social Affairs, Sustainable Development Goals
https://sdgs.un.org/goals

6 Membership of the Advisory Groups for the Subject Benchmark Statement for Mathematics, Statistics and Operational Research

Membership of the Advisory Group for the Subject Benchmark Statement for Mathematics, Statistics and Operational Research (2023)

Professor Duncan Lawson (Chair)	Coventry University
Dr Peter Rowlett (Deputy Chair)	Sheffield Hallam University
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Dr Ben Dias	easyJet
Professor Noel-Ann Bradshaw	University of Greenwich
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Dr Martyn Parker	University of Warwick
Dr Ewan Russell	University of Liverpool
Professor Chris Sangwin	University of Edinburgh
Dr Calvin James Smith	University of Reading
Dr Antonia Wilmot-Smith	University of St Andrews

Membership of the review group for the Subject Benchmark Statement for Mathematics, Statistics and Operational Research (2019)

The fourth edition, published in 2019, was revised by QAA to align the content with the revised UK Quality Code for Higher Education, published in 2018. Proposed revisions were checked and verified by the Chair of the Subject Benchmark Statement for Mathematics, Statistics and Operational Research review group from 2016.

Professor Duncan Lawson	Coventry University
Dr Alison Felce	QAA

Membership of the review group for the Subject Benchmark Statement for Mathematics, Statistics and Operational Research (2014)

Details provided below are as published in the third edition of the Subject Benchmark Statement.

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QAA officers	
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Membership of the benchmarking group for the annex to the Subject Benchmark Statement for Mathematics, Statistics and Operational Research (2009)

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Professor Adrian Bowman	Royal Statistical Society
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Dr Stuart Johns	Operational Research Society
Professor Duncan Lawson (Chair)	Higher Education Academy Subject Centre for mathematics, statistics and operational research
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Membership of the original benchmark statement group for mathematics, statistics and operational research (2002)

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Mr Gerald Goodall	The Royal Statistical Society
Mr Tony Palmer	De Montfort University
Professor Chris Robson (Chair)	University of Leeds
Dr Stephen Ryrie	University of the West of England, Bristol
Professor Peter Saunders	King's College London
Dr Stephen Siklos	University of Cambridge
Professor Joan Walsh	University of Manchester (retired)

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